You are about to embark on a great journey through the study of Limits, derivatives, and integrals!! We will hit a few small icebergs but nothing that will cause use to sink like the Titanic!! Enjoy your summer (with a little help from your math summer homework)! I cannot wait to see you all in the fall! - Kleinke

You should read ALL the information below ASAP!!!

What is AP Calculus All About?

We will cover all topics in the AP Calculus AB topic outline as they appear in the AP Calculus Course Description. The major textbook is Calculus – Graphical, Numerical, Algebraic by Finney et. Al. Of course my main objective for us is do well on the AP exam but I also want you to become better problem solvers using mathematics and be able to explain solutions to each other. I also want you to adapt to using a graphing calculator. My goal for you is to learn to appreciate the power of calculus and to give you the tools necessary to be successful in future math classes. You will work harder than you ever have before but I am here to help you and we will work together to succeed in calculus.

Requirements for AP Calculus:

- Graphing Calculator (with you daily)
- Binder with Tabs – not optional 1 in, plastic cover on front and back is helpful!
- Pencil
- Summer homework due First Day
- Highlighter and Colored Checking Pens

Summer Homework Expectations ....

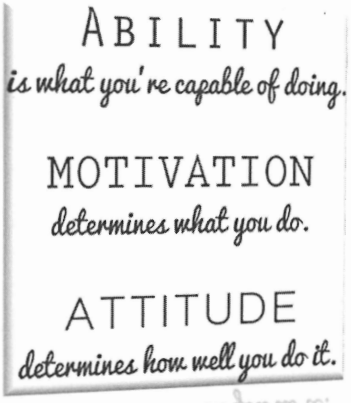
(1) Sign-up for Remind: send a text to 81010 with the message @24calc25
(2) Complete all questions USING the provided space. Any question without work earns no credit. ***PENCIL ONLY!!!!***
(3) Check all answers using the provided answers using a **different color pen.** Attempt to make corrections in the different color using the provided answer.
(4) Come to school on the first day ready to turn in the summer homework (already stapled and name on it)
(5) Come to school on the second day ready to ask a ton of questions!

Summer Homework Grading!

1) Collected on the first day of school
2) Checked in for completeness and to see if you checked your answers using a different color. Pencil for you doing the assignment, pen for checking! No Exception! I do not expect you to get them all right!

   The purpose of the summer homework is to refresh your memory on important skills learned in the past! It does none of us any good to copy or look up all the work to these problems!
3) Quiz on the material covered the first week of school!

4) All problems are given in worksheet form. Extra copies of this homework can be found on the high school homepage.
5) All answers should be rounded to 3 decimals or written as fractions (no mixed #)
6) All intervals should be given in INTERVAL notation.
7) Graded for completion, following directions, correcting work, and being on time.



An important part of the AP Calculus test is being able to follow directions. This really is an important part of life. Please make sure you follow directions (highlighting answers as requested, writing them in the correct notation, using a pencil, etc.)! All of these items will affect your score both on this assignment and on the AP Calculus Test in May.

If you give a decimal, the FINAL answer must be accurate to 3 decimal places. When giving an answer as a radical always reduce. When giving an answer as a fraction always reduce and always use improper fractions (no mixed numbers).

Time to get started!! Good Luck!!

Special Bonus Material!

At the end of this summer homework is the first unit of Notes that we will be covering (only half of it). Please detach these pages from the summer homework and place them into your binder! You can use them as reference as you complete the limit questions in this summer homework. (We covered all of this material in Honors Precalculus during the month of may!)

Name: _____ AP Calculus Summer Homework 2024

Directions: Please complete all work IN THE SPACE PROVIDED!! Write your answers on the given lines or IF no line is given then highlight your answer (yellow)!

1. Perform the indicated option for the following function notations.

   a. $f(x) = x - 2$, $g(x) = x + 1$ find $f(-6) + g(-6) =$ _____

   b. $h(x) = 3x - 5$, $g(x) = -2x^2 + 2x$ find $h(g(x-1)) =$ _____

2. Reduce each of the following as much as possible. Highlight your final answer in Yellow!!

   a. $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2\left(\frac{b^{3/2}}{a^{1/2}}\right)$

   b. $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

3. Solve each of the following problems. Don't forget to check for extraneous solutions. A calculator can be used but not the graphing feature. All of these should be done with work, algebraically! Highlight your final answer(s)!!!

   a. $12^{x-1} + 8 = 42$

   c. $|-10x - 9| = 69$

b. $\left(\frac{1}{27}\right)^{-x-2} = 243^{3x}$

d. $|8x - 4| = 36$

4. Given the function f(x) below, find the requested values.

a. $f(x) = \begin{cases} x^2 + 2, x \leq 1 \\ 2x^2 + 2, x > 1 \end{cases}$

f(-2) = _____

f(1) = _____

f(2) = _____

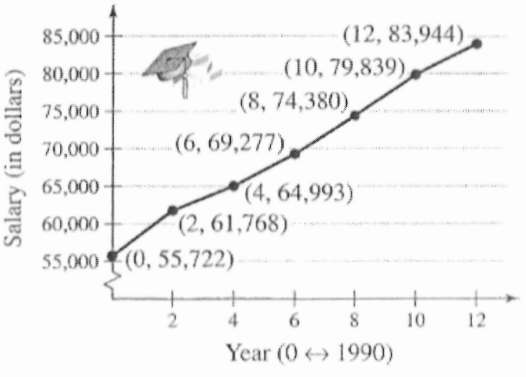
b. $f(x) = \begin{cases} 3x - 1, x < -1 \\ 4, -1 \leq x \leq 1 \\ x^2, x > 1 \end{cases}$

f(-2) = _____

f(-½) = _____

f(3) = _____

5. The graph given below shows the average salaries for senior high school principals from 1990 through 2002.



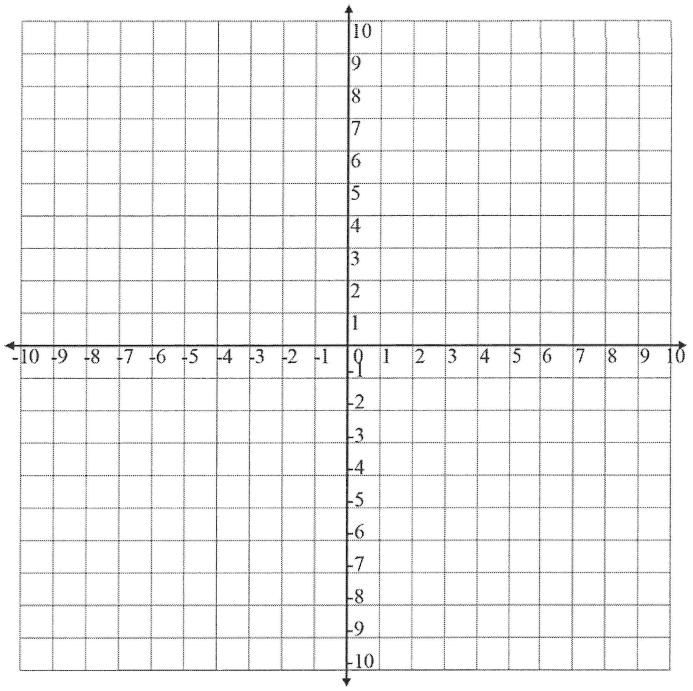
Calculate the average rate of change of salaries for senior high school principals over the first decade. Include proper units.

6. Given the function $f(x)=\begin{cases}1, x<0\\ \sqrt{x}, x\geq 0\end{cases}$:

a. Graph the function on the axes provided.

b. Determine the domain using your graph.

c. Determine the range using your graph.

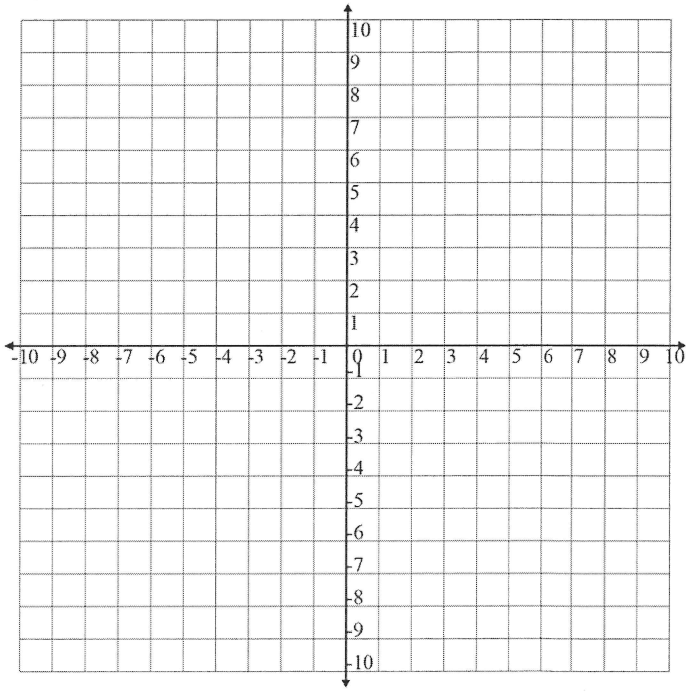


7. Given the function $g(x)=\begin{cases}x^2, x<0\\ x^3, 0\leq x\leq 1\\ 2x-1, x>1\end{cases}$:

a. Graph the function on the axes provided.

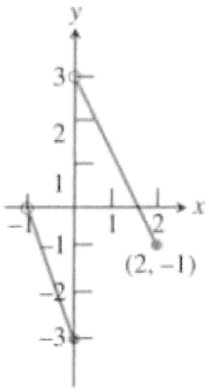
b. Determine the domain using your graph.

c. Determine the range using your graph.

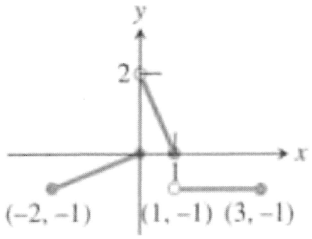


8. Write a piece-wise function for the given graphs below. Remember you need to give a function and the interval it covers. Your functions need to be in order based on their x-values from smallest to biggest.
Hint: When writing a piece-wise function write the function on each "piece" using what you know from algebra (slope, slope-intercept form, etc), then decide what interval you should assign to the x-values for that piece.

a.



b.



a

b

9. Find each of the following limits, then write your answer on the lines given at the right. Remember that NOTATION is SUPER important!!

a. $\lim_{x\to 4}\left(\dfrac{x-4}{x^2-3x-4}\right)$ a. _____

b. $\lim_{x\to 2}\left(4x^2+3\right)$ b. _____

c. $\lim_{x\to -3}\left(\dfrac{x^2+x-6}{x+3}\right)$ c. _____

d. $\lim_{x\to -5}\left(\dfrac{\sqrt{4-x}-3}{x+5}\right)$ d. _____

e. $\lim_{x\to 1}\left(\dfrac{x^2-1}{x-1}\right)$ e. _____

10. For each of the following limits, then write your answer on the lines given at the right. Remember that NOTATION is SUPER important!!

a. $\lim_{x\to\infty}\left(\frac{2x-5+4x^2}{3-5x+x^2}\right)$ a. _____

b. $\lim_{x\to\infty}\left(\frac{7x+6-2x^3}{3+14x+x^2}\right)$ b. _____

c. $\lim_{x\to\infty}\left(\frac{2x-5}{3-5x+3x^2}\right)$ c. _____

d. $\lim_{x\to 8}\left(\frac{x^2-64}{x-9}\right)$ d. _____

e. $\lim_{x\to 2}\left(\frac{x-2}{\sqrt{x}-\sqrt{4-x}}\right)$ e. _____

11. For each of the following limits, then write your answer on the lines given at the right. Remember that NOTATION is SUPER important!!

a. $\lim_{x\to 0}\left(\frac{\sin 6x}{x}\right)$ a. _____

b. $\lim_{x\to 2}\left(\frac{x-2}{x^3-4x}\right)$ b. _____

c. $\lim_{x\to \pi/4}\left(\frac{\sin x-\cos x}{\tan x-1}\right)$ c. _____

d. $\lim_{x\to 0}\left(\frac{\tan 4x}{9x}\right)$ d. _____

12. For each of the following limits, then write your answer on the lines given at the right. Remember that NOTATION is SUPER important!!

a. $\lim_{x\to 0}\left(\frac{(1+x)^2-1}{x}\right)$

b. $\lim_{x\to 0}\left(\frac{\sin^2 x}{x}\right)$

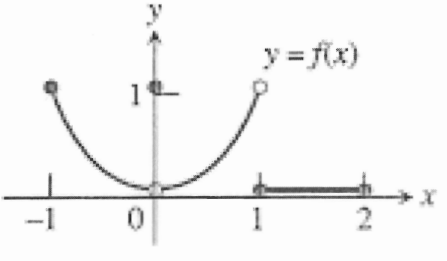
c. $\lim_{h\to 0}\left(\frac{\sqrt{4+h}-2}{h}\right)$

a. _____

b. _____

c. _____

13. Decide if the following are true or false!



(a) $\lim_{x\to -1^+} f(x) = 1$

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0^-} f(x) = 1$

(d) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

(e) $\lim_{x\to 0} f(x)$ exists

(f) $\lim_{x\to 0} f(x) = 0$

(g) $\lim_{x\to 0} f(x) = 1$

(h) $\lim_{x\to 1} f(x) = 1$

(i) $\lim_{x\to 1} f(x) = 0$

(j) $\lim_{x\to 2^-} f(x) = 2$

a. _____ f. _____

b. _____ g. _____

c. _____ h. _____

d. _____ i. _____

e. _____ j. _____

14. Factor each of the following questions as much as possible.

a. $3x^2 - 12x^3 =$ _____

b. $3x^2 + 10x - 8 =$ _____

c. $24xe^x - 3x^2e^x =$ _____

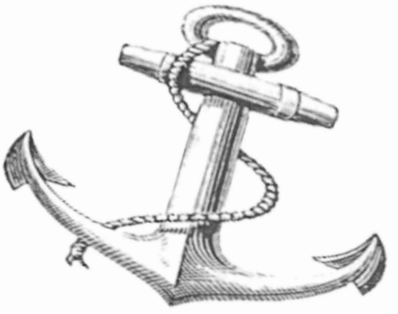
d. $x^2 - 36 =$ _____

e. $x^2 - 4x - 12 =$ _____

f. $x^3 - 8 =$ _____

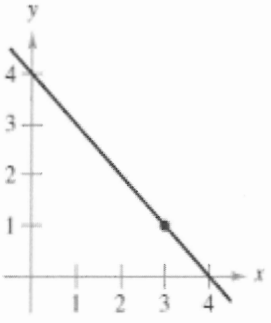
g. $x^6 - 16x^4 =$ _____

h. $8x^3 + 27 =$ _____

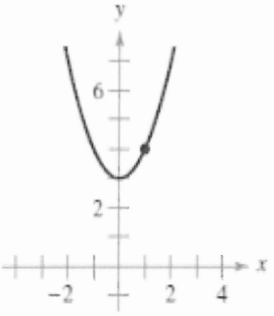


In Problems 15 – 22, use the graph to find the limit (if it exists). HIGHLIGHT YOUR FINAL ANSWER FOR EACH PROBLEM!

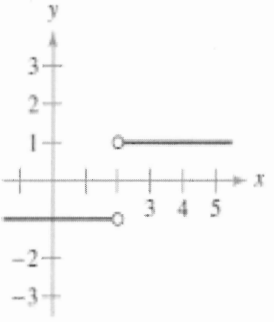
15. $\lim_{x \to 3} (4 - x)$



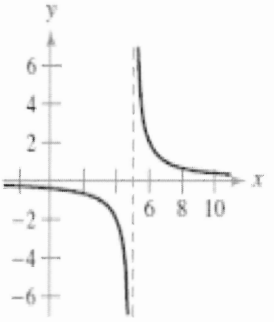
16. $\lim_{x \to 1} (x^2 + 3)$



19. $\lim_{x \to 2} \dfrac{|x - 2|}{x - 2}$

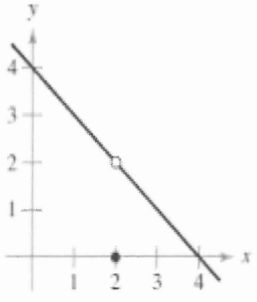


20. $\lim_{x \to 5} \dfrac{2}{x - 5}$



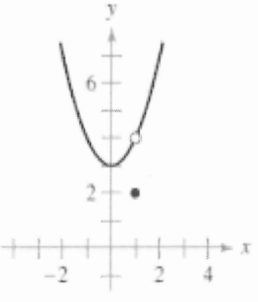
17. $\lim_{x \to 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

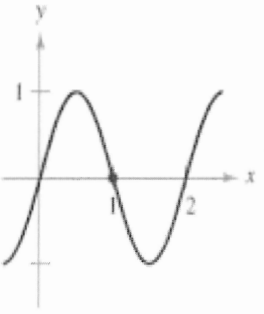


18. $\lim_{x \to 1} f(x)$

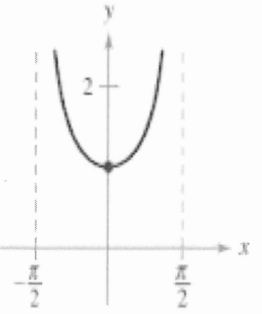
$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



21. $\lim_{x \to 1} \sin \pi x$



22. $\lim_{x \to 0} \sec x$



23. These questions should be extremely quick! You should have all of these memorized or done quickly using the hand-trick.

a. $\sin\left(\frac{\pi}{2}\right) =$ _____

b. $\sin\left(\frac{\pi}{4}\right) =$ _____

c. $\sin\left(\frac{\pi}{3}\right) =$ _____

d. $\sin(0) =$ _____

e. $\sin\left(\frac{\pi}{6}\right) =$ _____

f. $\cos\left(\frac{\pi}{2}\right) =$ _____

g. $\cos\left(\frac{\pi}{4}\right) =$ _____

24. Basic exponents rules and logs are very important. You should have these three memorized!

a. $\ln e =$ _____ b. $\ln 0 =$ _____ c. $e^{\ln x} =$ _____

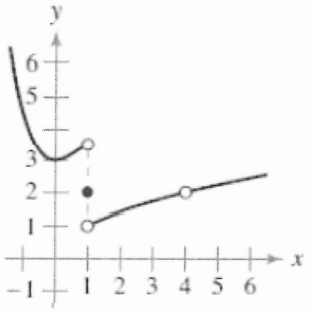
In Problems 25 & 26, use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it.

25. (a) $f(1)$

(b) $\lim_{x\to 1} f(x)$

(c) $f(4)$

(d) $\lim_{x\to 4} f(x)$



(a) _____ (c) _____

(b) _____ (d) _____

26. (a) $f(-2)$

(b) $\lim_{x\to -2} f(x)$

(c) $f(0)$

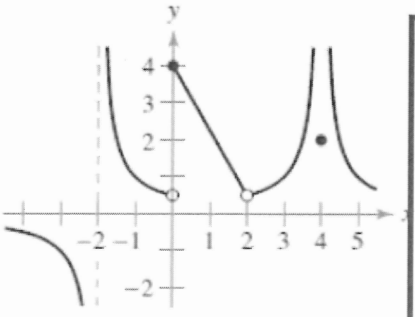
(d) $\lim_{x\to 0} f(x)$

(e) $f(2)$

(f) $\lim_{x\to 2} f(x)$

(g) $f(4)$

(h) $\lim_{x\to 4} f(x)$



(a) _____ (e) _____

(b) _____ (f) _____

(c) _____ (g) _____

(d) _____ (h) _____

Use the following answers to check your work! Make corrections if possible, otherwise come with questions the first week of school!

1a. -13

1b. $-6x^2 + 18x - 17$

2a. $a^{5/6}b^{1/2}$

2b. $\frac{5x}{x+5}$

3a. 2.419

3b. ½

3c. -7.8, 6

3d. 5, -4

4a. 6, 3, 10

4b. -7, 4, 9

5. 2411.7

6b. $(-\infty, +\infty)$

6c. $[0, +\infty)$

7b. $(-\infty, +\infty)$

7c. $[0, +\infty)$

8a. $f(x) = \begin{cases} -3x - 3, -1 < x \leq 0 \\ -2x + 3, 0 < x \leq 2 \end{cases}$

8b. $f(x) = \begin{cases} 0.5x, -2 \leq x \leq 0 \\ -2x + 2, 0 < x \leq 1 \\ -1, 1 < x \leq 3 \end{cases}$

9a. 1/5

9b. 19

9c. -5

9d. -1/6

9e. 2

10a. 4

10b. DNE, $-\infty$

10c. 0

10d. 0

10e. $\sqrt{2}$

11a. 6

11b. 1/8

11c. $\frac{\sqrt{2}}{2}$

11d. 4/9

12a. 2

12b. 0

12c. ¼

13a. True

13b. True

13c. False

13d. True

13e. True

13f. True

13g. False

13h. False

13i. False

13j. False

14a. $3x^2(1-4x)$

14b. $(3x - 2)(x + 4)$

14c. $3xe^x(8 - x)$

14d. $(x + 6)(x - 6)$

14e. $(x + 2)(x - 6)$

14f. $(x - 2)(x^2 + 2x + 4)$

14g. $x^4(x - 4)(x + 4)$

14h. $(2x + 3)(4x^2 - 6x + 9)$

15. 1

16. 4

17. 2

18. 4

19. DNE

20. DNE

21. 0

22. 1

23a. 1

23b. $\frac{\sqrt{2}}{2}$

23c. $\frac{\sqrt{3}}{2}$

23d. 0

23e. ½

23f. 0

23g. $\frac{\sqrt{2}}{2}$

24a. 1

24b. DNE

24c. x

25a. 2

25b. DNE

25c. DNE

25d. 2

26a. DNE

26b. DNE

26c. 4

26d. DNE

26e. DNE

26f. ½

26g. 2

26h. DNE, $+\infty$

Questions & thoughts

Unit 1: Limits & Continuity

Topic 1: Introducing Calculus: Can Change Occur at an Instant

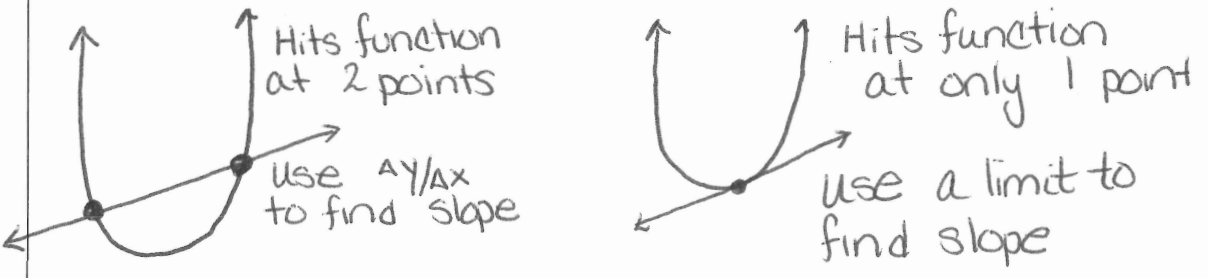
Words that Mean the Same Thing:

Average Velocity = Slope of the Secant line → Old-fashion Slope

Instantaneous Velocity = Slope of the Tangent Line → Need a Limit

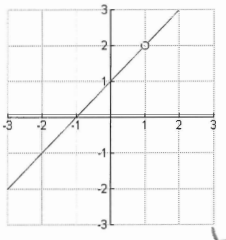
Diagrams:

Secant Line vs. Tangent Line



Topic 2: Defining Limits & Using Limit Notation

Definition of Limit:

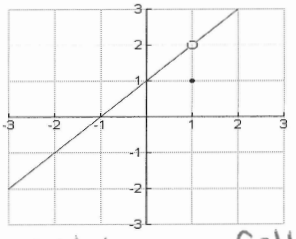


* Notice that the hole does not affect limit

$$f(x)=\frac{x^2-1}{x-1}$$

$\lim_{x\to 1} f(x) = 2$

$f(1)$ DNE

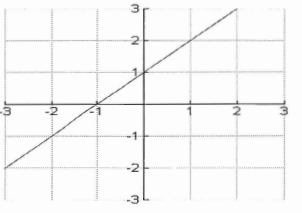


* Hole is filled but NOT on line - No Affect on Limit

$$g(x)=\begin{cases}\dfrac{x^2-1}{x}, x\neq 1\\ 1\ , x=1\end{cases}$$

$\lim_{x\to 1} g(x) = 2$

$f(1) = 1$



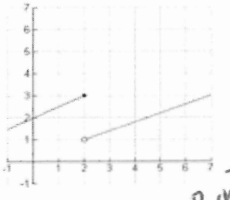
$$h(x)=x+1$$

$\lim_{x\to 1} h(x) = 2$

$f(1) = 2$

Topic 3: Estimating Limit Values from Graphs

One-Sided Limits

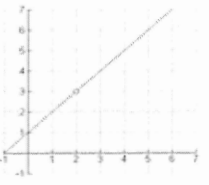


$\lim_{x\to 2^-} f(x) = 3$ (from left)

$\lim_{x\to 2^+} f(x) = 1$

$\lim_{x\to 2} f(x)$ = DNE (from Right)

↑ Left must Equal Right to exist

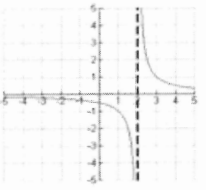


$\lim_{x\to 2^-} f(x) = 3$

$\lim_{x\to 2^+} f(x) = 3$

$\lim_{x\to 2} f(x) = 3$

Questions & thoughts



$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = +\infty$

$\lim_{x\to 2} f(x) = DNE$

Topic 4: Estimating Limit Values from Tables

Example: Using the numerical (Table) Method Solving the following limit.

$$\lim_{x\to 9}\left(\frac{x-9}{\sqrt{x}-3}\right)$$

Topic 5: Determining Limits Using Algebraic Properties of Limits

Remember all the properties of limits we talked about last year!

* Always try Direct Sub first.
* Notation matters

Examples: $\lim_{x\to -2}(2x-2)$ $= -6$

$\lim_{x\to 0}\left(\sqrt{x+1}\right)$ $= 1$

$\lim_{x\to -5}\left(\frac{x^2+6x+5}{x+5}\right)$

$\lim_{x\to -5}\frac{(x+5)(x+1)}{x+5}$

$\lim_{x\to -5}(x+1)$

$= -5+1$

$= -4$

Example:

(a) $f(-3/2) = DNE$

(b) $f(2) = 4$

(c) $f(3/2) = 10$

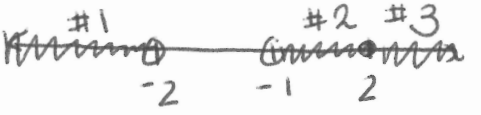
(d) $\lim_{t\to -2} f(t) = 4$

(e) $\lim_{t\to -1^+} f(t) = 5/2$

(f) $\lim_{t\to 2} f(t) = 4$

(g) $\lim_{t\to 0} f(t) = \frac{6}{0}$ DNE

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \quad (1) \\ \dfrac{t+6}{t^2-t} & \text{for } -1 < t < 2 \quad (2) \\ 3t-2 & \text{for } t \geq 2 \quad (3) \end{cases}$$



Helpful way to decide which function to use

Questions & thoughts

Topic 6: Determining Limits Using Algebraic Manipulation

There are many different algebra techniques you can do to manipulate an expression to help find a limit algebraically. These techniques include:

1. Factoring *2. Distributing* *3. Multiplying by the conjugate*

4. Finding a Common Denominator *5. Reducing Trig Expressions with IDs*

Find the limit of each of these expressions below.

Example 1: $\lim_{x\to\pi/2}\left(\frac{\tan x}{\sec x}\right)$ $= \lim_{x\to\pi/2} \frac{\sin x/\cos x}{1/\cos x}$

$\tan\frac{\pi}{2}$ = DNE so change to sines & cosines

$= \lim_{x\to\pi/2} \frac{\sin x}{\cos x}\cdot\frac{\cos x}{1}$ * USE HAND-TRICK

$= \lim_{x\to\pi/2} \sin x = \sin \pi/2 = 1$

Example 2: $\lim_{x\to 4}\left(\frac{\sqrt{x}-2}{x-4}\right)$

Can either Rationalize or factor to solve

$= \lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)}$

$= \lim_{x\to 4} \frac{1}{\sqrt{x}+2}$

$= \frac{1}{4}$

OR

$\lim_{x\to 4} \frac{(\sqrt{x}-2)}{(\sqrt{x}-2)(\sqrt{x}+2)}$

$\lim_{x\to 4} \frac{1}{\sqrt{x}+2}$

$= 1/4$

Example 3: $\lim_{x\to 5}\left(\frac{x-5}{\sqrt{x+4}-3}\right)\cdot\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$

$= \lim_{x\to 5} \frac{(x-5)(\sqrt{x+4}+3)}{x+4-9}$

$= \lim_{x\to 5} \frac{(x-5)(\sqrt{x+4}+3)}{x-5}$

$= \lim_{x\to 5} \sqrt{x+4}+3$

$= \sqrt{9}+3$

$= 6$

Example 4: $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$

$= \lim_{x\to 1} \frac{x+1-2}{x^2-1}$

$= \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)}$

$\lim_{x\to 1} \frac{1}{x+1}$

$= 1/2$

Questions & thoughts

Example 5: $\lim_{x\to 2}\left(\frac{x^2-x+5}{x-2}\right) = \frac{7}{0}$ DNE

Not factorable so cannot be Reduced.

Example 6: $\lim_{x\to 0}\left(\frac{(x+a)^2-a^2}{x}\right)$

$= \lim_{x\to 0} \frac{x^2+2xa+a^2-a^2}{x}$

$= \lim_{x\to 0} \frac{x^2+2xa}{x}$

$= \lim_{x\to 0} \frac{x(x+2a)}{x}$

$= \lim_{x\to 0} x+2a$

$= 2a$

MEMORIZE: $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1$ $\lim_{x\to 0}\left(\frac{1-\cos x}{x}\right) = 0$

* Remember $\sin ax$ can't factor the "a" out

Example 7: $\lim_{x\to 0}\left(\frac{\sin 4x}{x}\right)\cdot\frac{4}{4}$

$= 4\lim_{x\to 0} \frac{\sin 4x}{4x}$

$= 4(1)$

$= 4$

Example 8: $\lim_{x\to 0}\left(\frac{\csc 8x}{\csc 4x}\right)$

$= \lim_{x\to 0} \frac{1/\sin 8x}{1/\sin 4x}$

$= \lim_{x\to 0} \frac{\sin 4x}{\sin 8x}$

$= \lim_{x\to 0} \frac{\sin 4x}{1x}\cdot\frac{1x}{\sin 8x}$

$= \lim_{x\to 0} \frac{\sin 4x}{x}\cdot\lim_{x\to 0}\frac{x}{\sin 8x}$

$\lim_{x\to 0} \frac{\sin 4x}{x}\cdot\frac{4}{4}\cdot\lim_{x\to 0}\frac{x}{\sin 8x}\cdot\frac{8}{8}$

$4\lim_{x\to 0} \frac{\sin 4x}{4x}\cdot\frac{1}{8}\lim_{x\to 0}\frac{8x}{\sin 8x}$

$= 4(1)(1/8)(1)$

$= 1/2$